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EXPERIMENTAL ANALYSIS OF COMBINED ACTION OF BENDING, SHEAR AND TORSION ON TIMBER BEAMS

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Preliminary notes

Wood is an anisotropic material, i.e., it is composed of fibres that correspond better to normal stresses in the longitudinal direction or in the direction of fibres and poorly to stresses perpendicular to the fibres and to the longitudinal shear stresses. In practical situations it is better to avoid these stresses, although that is often very difficult. Because of this, the analysis of different interactions between shear and normal stresses perpendicular to the fibres is necessary. In this paper an experimental analysis of interaction of stresses from torsion and bending caused by an eccentric shear force was performed. The results are compared in a simple and practical way with the calculated values and the values from a linear numerical model. The paper analysed shear stresses parallel to the fibres and normal stresses perpendicular to the fibres and the results are shown numerically and by a force/displacement diagrams.

Keywords: *bending, experimental analysis, interaction, shear, stress, timber beam, torsion, wood*

Eksperimentalna analiza zajedničkog djelovanja savijanja, posmika i torzije drvenih nosača

Prethodno priopćenje

Drvo je anizotropan materijal, odnosno, sastoji se od skupa vlakana koja bolje podnose normalna naprezanja u svom uzdužnom pravcu ili pravcu vlakana, a slabije okomito na pravac vlakana te posmično u uzdužnom pravcu. U praktičnim slučajevima potrebno je izbjegavati pojavu ovih nepovoljnih naprezanja, iako je to u pravilu teško izvedivo. Iz tog razloga, neizbježne su analize interakcija između svih posmičnih te normalnih naprezanja. U radu je eksperimentalno analizirana interakcija naprezanja od torzije i savijanja kroz djelovanje ekscentričnom poprečnom silom, a rezultati su na jednostavan i praktičan način uspoređivani s računskim vrijednostima te vrijednostima dobivenim linearnim numeričkim modelom. Naglasak je stavljen na posmična naprezanja paralelno vlaknima i normalna naprezanja okomito na vlakna, a rezultati su prikazani tablicom i dijagramima sila/pomak.

Ključne riječi: *drveni nosač, drvo, eksperimentalna analiza, interakcija, naprezanje, posmik, savijanje, torzija*

1 Introduction

Although shear strength, after tensile strength perpendicular to the fibres, is the smallest strength of the wood, the design of timber structures for the influence of shear according to eurocode EC5 [1] rarely dictates the final size of the elements cross-section. The shear stresses can be calculated according to [2] as:

$$\tau_{xz} = \frac{T_z S_y}{b I_y}; \quad \tau_{xy} = \frac{T_y S_z}{b I_z}. \tag{1}$$

For the design of wooden beams to normal and shear stresses, according to [3], the simplified expressions can be used:

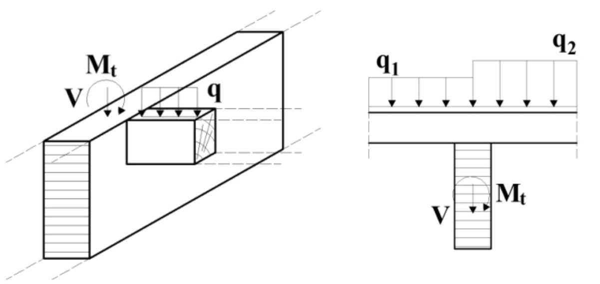
$$\sigma_{x,x} = \sigma_{\mathrm{m},y,\mathrm{d}} = \pm \frac{M_{y,\mathrm{d}}}{W_y} \le f_{\mathrm{m,d}}, \tag{2}$$

$$\tau_{x,z} = \tau_{\mathrm{v,d}} = 1{,}5 \frac{V_\mathrm{d}}{A} \le f_{\mathrm{v,d}}. \tag{3}$$

Most commonly shear force has its greatest value in the proximity of the supports, where it is also common to have a reduction of the cross-section in order to reduce the amount of used material, which then results in a complex state of stresses. The analysis of this state of stresses, whose main components are shear and tensile stresses perpendicular to the fibres, is done through the design of shear stress with the reduction of shear strength by a factor k_v (caused by the reduction of the cross section in the lower zone) and the reduction of shear force in the proximity of the supports. Although these places in structures are dangerous and require additional attention in shear design, the shear stresses rarely exceed the allowed values. Beside this problem, in shear design, the influence of cracks [4] is taken into account also by reducing the shear strength with factor k_cr. Shear stresses also appear because of the presence of torsion, and according to [2] we use these expressions:

$$\tau_{xz} = \frac{M_\mathrm{t}}{W_\mathrm{tor}}; \quad W_\mathrm{tor} = \alpha h b^2; \quad \tau_{xy} = \eta \tau_{xz}. \tag{4}$$

Coefficient α is used for torsion of rectangular cross sections by defining the section modulus (W_tor) from the ration of the height (h) and the width (b). Coefficient η defines the relation between the shear stress on longer (h) and shorter (b) side of the cross section.



a) Beams at the end where secondary beams come only from one side b) Non symmetrical loading of a beam

Figure 1 Examples of beams where the interaction occurs

Through the design of wooden structures we try to avoid torsion of elements with different shape and constructive measures, and carry all loads through preferably bending. If we can succeed in doing that we will have only tensile and/or compressive normal stresses to which wood corresponds better. In practice it is rarely possible to have pure bending and/or torsion and not to have shear stresses in the fibre direction and normal tensile stresses perpendicular to the fibres. Also it is rarely possible to avoid different imperfections during construction or similar deviations from planed shapes and sizes, which will result in torsion and other non favourable stresses. The case of interaction of bending, shear and torsion can occur on roof beams and frames as shown in Fig. 1.

One of the preliminary versions of eurocodes and literature that is based on them [3], supplied an approximate design procedure for the interaction of shear and torsion, and in that design the emphasis was made on the influence of torsion over shear. The expression is:

$$\frac{\tau_{\mathrm{tor,d}}}{f_{\mathrm{tor,d}}}+\left(\frac{\tau_{\mathrm{v,d}}}{f_{\mathrm{v,d}}}\right)^2 \leq 1, \tag{5}$$

where it was proposed to use the shear strength of wood $f_{\mathrm{v,d}}$ for the torsion strength $f_{\mathrm{tor,d}}$.

According to the earlier used design procedures, Method of allowed stresses [5], the following expression was used:

$$\tau_{\mathrm{II}}=\tau_{\mathrm{t}}+\tau_{\mathrm{m}} \leq \tau_{\mathrm{IId}}, \tag{6}$$

where the longitudinal shear stress from torsion and bending is summed up and compared to the allowed stress.

Bearing in mind that wood is an anisotropic material consisting of many "tubes" (tracheas) [6] and [7], that correspond better to loads in longitudinal direction or in the direction of the fibres, and poorly to loads perpendicular to the fibres, we can assume that the logical and probable collapse from bending and/or torsion will be initiated with the detachment of tracheas. That means that the element would break under exceeded longitudinal shear and/or normal tensile stress perpendicular to the fibres. Along with the relation between different stresses, the moisture content in wood is important and is still studied by many authors [8]. In this investigation only one kind of solid timber was used and all specimens were kept in the same conditions, i.e., they had the same moisture content.

2 Experimental analysis

Geometry of the used beam, characteristics of the material, characteristic stresses with the assumed material strengths are shown in Tab. 1.

In the experiment the beam was loaded with an eccentric force (with a constant eccentricity ev = 8 cm) and the deformations were measured at the border fibres in the middle of the span. A schematic overview of the load application, points and direction of deformation measurements are shown in Fig. 2, and the boundary and load applying conditions are shown in Figs. 3, 4 and 5.

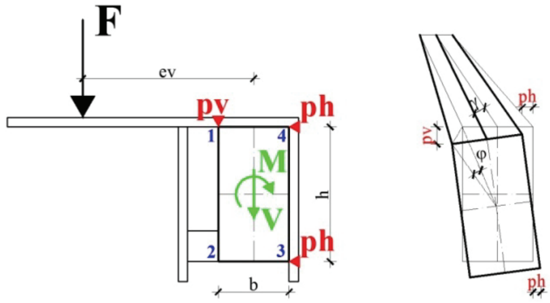


Figure 2 Schematic view of load applying and measured deformation

The beam was firstly subjected to a series of smaller loads and unloads, and after that the breaking load was applied ($F \approx 2100$ N, $V_{\mathrm{stv}} \approx 1050$ N). The same procedure was performed on two more identical beams with the same geometry and boundary conditions. As can be seen this experiment produced bending, shear and torsion stresses and therefore its results present the interaction of mentioned stresses.

Figure 3 Testing machine

The failure of the tested beams occurred through a spiral longitudinal (fibre direction) crack, as shown in Figs. 6, 7 and 8. It can be assumed that the breaking happened from the influence of the combined shear stress in the longitudinal direction and normal tensile stress perpendicular to the fibres. Due to natural curvature of the fibres and sawing of wood some fibres do not extend throughout the entire beam. Because of this they cannot transfer stresses (load) further and at this point the crack begins. In this manner the cross section begins to weaken and break, as shown in Fig. 8.

Table 1 Characteristics of the tested beam

Geometry / mm	Assumed material characteristics of solid timber C14 / N/mm^2	Shear stress – bending by shear force / N/mm^2	Shear stress - torsion / N/mm^2	Total shear stress / N/mm^2	Tensile stress perpendicular to the fibres / N/mm^2
b = 23,0 h = 43,0 L = 554	$E_{0,\mathrm{mean}}$=7000 $E_{90,\mathrm{mean}}$=230 G_{mean}=440	$\tau_{\mathrm{v}} = 1{,}5V/A \leq f_{\mathrm{v}}$ f_{v} = 1,7 N/mm^2	$\tau_{\mathrm{tor}} = M_{\mathrm{tor}}/\alpha h b^2 \leq f_{\mathrm{v}}$ f_{v} = 1,7 N/mm^2 α = 0,242, β = 0,222	f_{v} = 1,7 N/mm^2	$f_{\mathrm{t,90}}$ = 0,3 N/mm^2

Figure 4 Measuring of deformation and applying of load

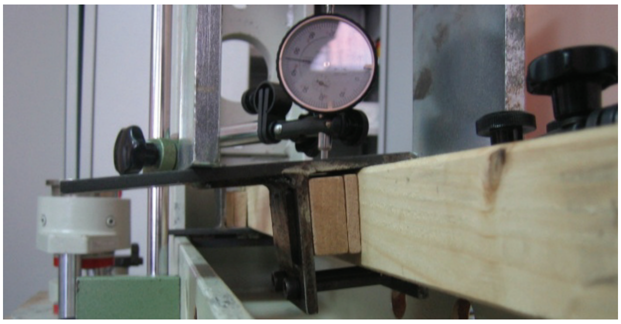


Figure 5 Deformation during the applying of load

Figure 6 The failure of the sample

According to the assumptions and in order to have a better insight into influence of torsion the measured vertical and horizontal deformations were transferred into the relative rotation angle around the longitudinal axis of the beam. The results are shown in the shape of the shear force/displacement diagrams, Fig. 9, and compared in Tab. 2 as radians and stresses. In the diagrams in Fig. 9 the thinner lines represent series of smaller loads and unloads and the thicker (blue, red and green) represent loading until the breaking of the specimen. All three specimens showed very similar results and for better overview only mean value is used in the diagrams.

Figure 7 Failure by separation of the ends of fibres

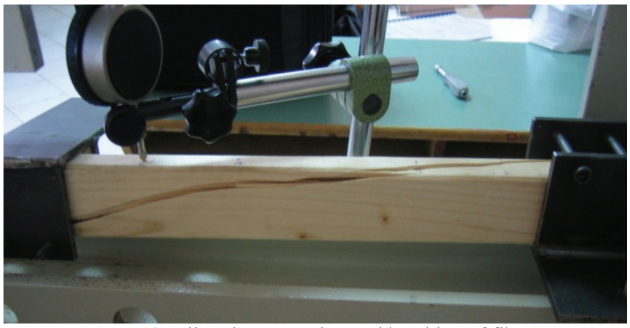


Figure 8 Failure by separation and breaking of fibres

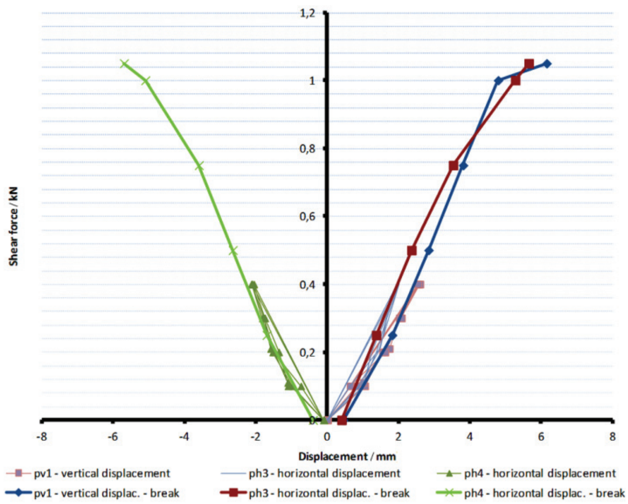


Figure 9 Force/deformation diagram

3 Numerical analysis - stress distribution

In order to have a better insight into the stress distribution on a three dimensional element, as all structures actually are, a finite element analysis was performed for all three cases: bending with shear force, torsion and the combined bending with shear force and torsion. The analysis was conducted for three values of forces from which the last one had a value approximately as the breaking force ($F \approx 2000$ N, $V_{stv} \approx 1000$ N) as shown in Tab. 2. Because the used beam represents a statically determined structure a linear finite element model with isotropic material was computed using software ROBOT [9]. The 4-node tetrahedrons finite elements were used and the boundary conditions were defined with fixed deformations at edges of the model in order to present fork-like pinned supports. Beams with the applied loads and inner forces are shown in Fig. 10. In order to simulate bending and shear (Fig. 10a) the load (F) is applied in the middle of the cross section. The torsion (Fig. 10b) is simulated by applying two forces on the edges of the cross section. Bending with shear and torsion (Fig. 10c) was simulated by an eccentric force on only one edge of the cross section. All these cases were also simulated in the FEM analysis (Figs. 11, 12 and 13).

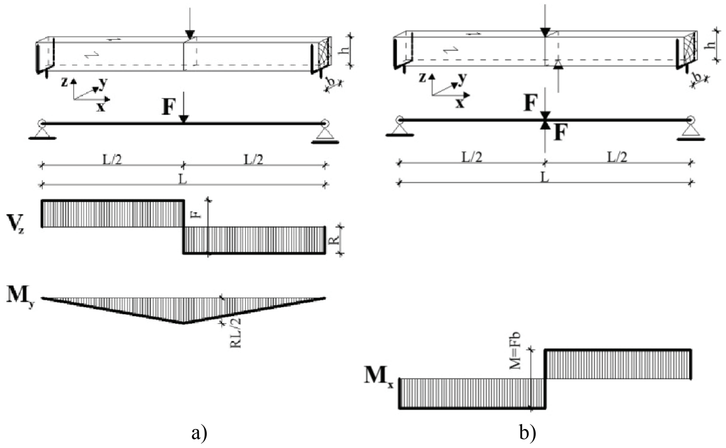
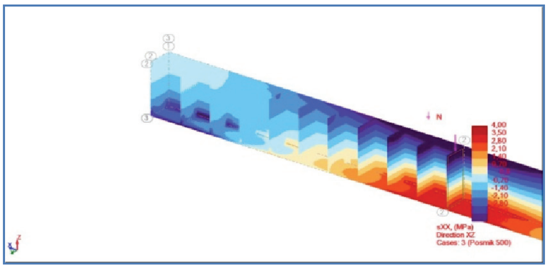
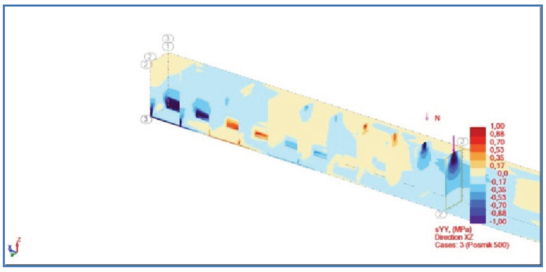


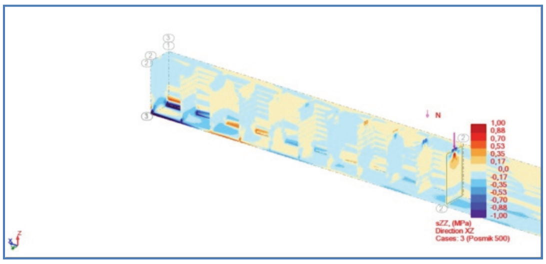
Figure 10 Examples of the applied load: a) Bending with shear, b) Torsion, c) Bending with shear and torsion



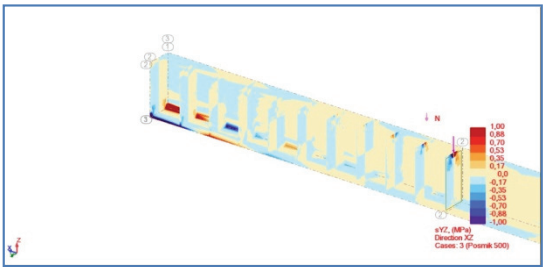
a) Normal stresses x-x parallel to the fibres



b) Normal stresses y-y perpendicular to the fibres



c) Normal stresses z-z perpendicular to the fibres



d) Shear stresses parallel to the fibres

Figure 11 The distribution of stress - bending with shear

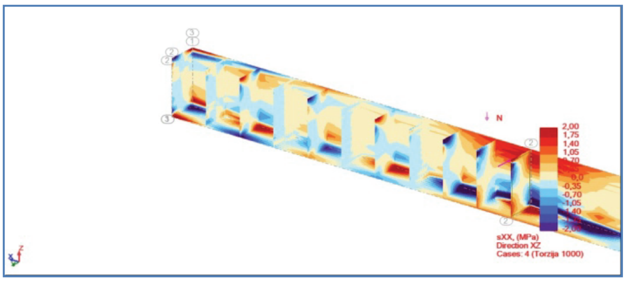
The distribution of normal and shear stresses on the selected model for all load cases: bending with shear force, torsion and combined bending with shear force and torsion is shown on the outer surface and on several sections in Figs. 11, 12 and 13.

It can be seen that the concentration of stresses occurs near the support (both perpendicular normal stresses and shear stresses) and that was confirmed with the results of the experiment, Figs. 6, 7 and 8, i.e., the failure of the specimen.

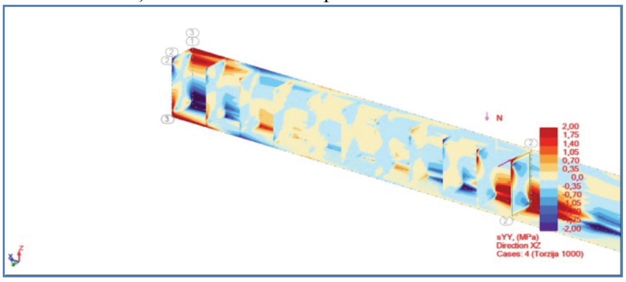
The stresses from torsion also exhibit similar points of concentration but a little closer to the support. The stress from the combination is a sum of the two previous cases.

4 Comparison of results

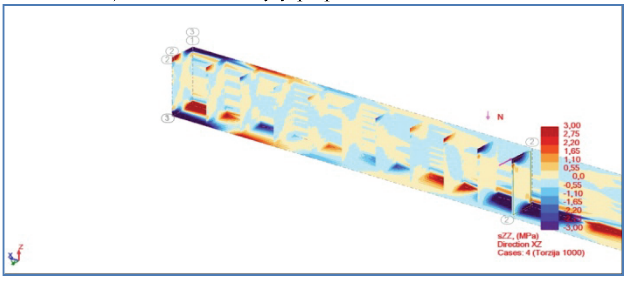
In Tab. 2 the calculated stresses, the stresses from the experiment and the finite element analysis results are compared.



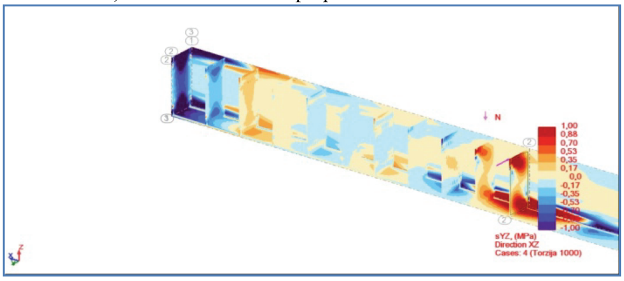
a) Normal stresses *x-x* parallel to the fibres



b) Normal stresses *y-y* perpendicular to the fibres

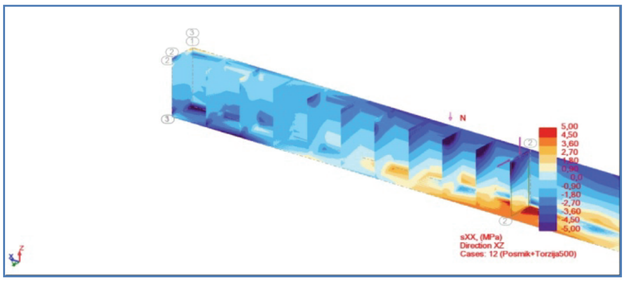


c) Normal stresses *z-z* perpendicular to the fibres

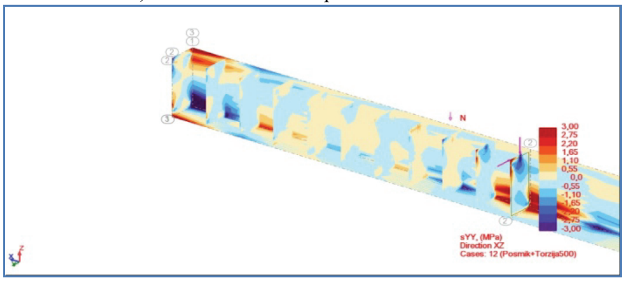


d) Shear stresses parallel to the fibres

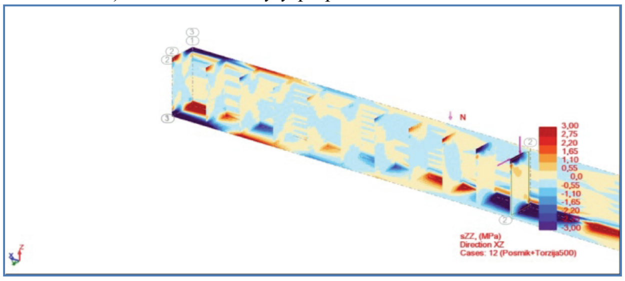
Figure 12 Distribution of stress - torsion



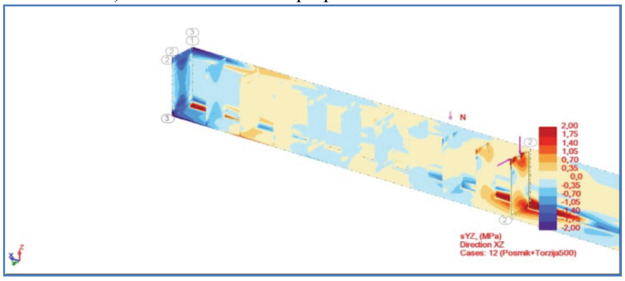
a) Normal stresses *x-x* parallel to the fibres



b) Normal stresses *y-y* perpendicular to the fibres



c) Normal stresses *z-z* perpendicular to the fibres



d) Shear stresses parallel to the fibres

Figure 13 Distribution of stress - bending with shear and torsion

Table 2 Comparison of results

Geometry / mm	Shear stress from bending with shear force / N/mm^2	Shear stress from torsion / N/mm^2	Shear stress from bending with shear force and torsion / N/mm^2	Read shear stress from bending with shear force and torsion / N/mm^2	Normal stress perpendicular to the fibres from bending with shear force and torsion / N/mm^2
	Calculation for experimental values	Experiment	Calculation + experiment	FEA	FEA
	$\tau_v = 1{,}5V_{stv}/A \leq 1{,}7$ N/mm^2	$\tau_{tor} = \varphi G \beta b / \alpha L \leq f_v$	$\tau_{uk} = \tau_{tor} + \tau_v \leq f_v$	$\tau_{uk} = \tau_{tor} + \tau_v \leq f_v$	$f_{t,90} = 0{,}3$ N/mm^2
$b = 23{,}0$ $h = 43{,}0$ $L = 554$	for $V_{stv} \approx 500$ N $\tau_v = 0{,}76 < 1{,}7$ for $V_{stv} \approx 750$ N $\tau_v = 1{,}14 < 1{,}7$ for $V_{stv} \approx 1000$ N $\tau_v = 1{,}52 < 1{,}7$	for $V_{stv} \approx 500$ N $\rightarrow$ $\varphi = 0{,}1698$ rad $\tau_{tor} = 0{,}545 < 1{,}7$ for $V_{stv} \approx 750$ N $\rightarrow$ $\varphi = 0{,}2388$ rad $\tau_{tor} = 0{,}767 < 1{,}7$ for $V_{stv} \approx 1000$ N $\rightarrow$ $\varphi = 0{,}3274$ rad $\tau_{tor} = 1{,}052 < 1{,}7$	for $V_{stv} \approx 500$ N $\tau_{uk} = 1{,}305 < 1{,}7$ for $V_{stv} \approx 750$ N $\tau_{uk} = 1{,}907 \approx 1{,}7$ for $V_{stv} \approx 1000$ N $\tau_{uk} = 2{,}572 > 1{,}7$	for $V_{stv} \approx 500$ N $\tau_{uk} = 1{,}285 < 1{,}7$ for $V_{stv} \approx 750$ N $\tau_{uk} = 1{,}818 \approx 1{,}7$ for $V_{stv} \approx 1000$ N $\tau_{uk} = 2{,}351 > 1{,}7$	for $V_{stv} \approx 500$ N $\sigma_{t,90,stv} = 0{,}18 < f_{t,90}$ for V$_{stv} \approx 750$ N $\sigma_{t,90,stv} = 0{,}32 > f_{t,90}$ for $V_{stv} \approx 1000$ N $\sigma_{t,90,stv} = 0{,}46 > f_{t,90}$

For three selected values of shear forces, as shown in Tab. 2, the shear stresses for all load cases were calculated and compared. The highest shear stress from bending with shear force was calculated and added to the shear stress from torsion calculated from the relative angle of rotation from the experiment. As can be seen the highest shear stress from bending with shear force does not exceed the shear strength, i.e., it does not cause the failure. Because of that it can be concluded that the influence of shear stress from torsion is what causes the failure.

The tensile normal stress perpendicular to the grain was calculated numerically and it can be noticed that it also exceeds the strength for the force similar to the force necessary to exceed the shear strength (Tab. 2).

5 Conclusion

On the basis of the experimental analysis of shear and normal stresses caused by shear forces, bending and torsion it is shown that the cause of the cracks and failure of beams are shear and normal tensile perpendicular stresses caused by torsion. The fact that wood is a natural anisotropic material with always present imperfections caused by manufacturing, building and specific situations of loading only emphasises this problem. Bearing this in mind it can be said that stresses from torsion cannot be avoided and that it is a necessity to check in more detail than present design checks. Also in present design checks there is no solution or way to investigate other shear stresses and/or combination of these with other normal (parallel and/or perpendicular) stresses which can be a topic for some future investigations.

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